

Economic policy in a complex world Problems

1. The Barro-Gordon model¹

Monetary policy aims at minimising a loss function given by:

$$\text{Min } L_t = \frac{1}{2}((y - k\bar{y})^2 + \omega\pi^2)$$

Where $k \geq 1$, y is the output and π is the inflation rate, subject to the following constraints:

$$y = \bar{y} - \beta(\pi^a - \pi)$$

where \bar{y} is the equilibrium output level and π^a is the expected rate of inflation. For simplicity it is supposed that monetary policy can directly determine the inflation rate.

1. Explain the model. What is the effect of unanticipated inflation? Why? What does $k > 1$ correspond to?
2. Suppose that monetary policy is decided after private agents have formed expectations. Show that inflation is then set at:

$$\tilde{\pi} = \frac{\beta}{\omega + \beta^2} (k - 1)\bar{y} + \frac{\beta^2}{\omega + \beta^2} \pi^a$$

Comment this result.

3. Suppose inflation expectations are adaptative, i.e. $\pi^a = \pi_{-1}$. Analyse graphically the evolution of inflation in a (π^a, π) schedule, assuming inflation expectations are initially nil.
4. Suppose now inflation expectations are rational, i.e. that agents know the loss function and form expectations accordingly. Determine equilibrium inflation and output first graphically, then analytically. Show that the policy outcome is worse than if the central bank had avoided engaging in loss minimization.
5. Discuss policy implications.

2. A bare-bones coordination model

Consider a two-country, symmetric model under fixed exchange rates. Countries each have one single instrument, namely fiscal policy. Starred variables represent the foreign country, non-starred ones the home country. Fiscal expansion in each country has an effect on its

¹ Barro, Robert, and David Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, Vol. 91, No. 4, pp. 589-610.

neighbor, so that, if Y represents production (measured as the gap between actual production and full employment) and g the fiscal instrument:

$$Y = \phi g + \psi g^* - u$$

$$Y^* = \phi g^* + \psi g - u$$

where $\phi > \psi > 0$ and u represents a symmetric external shock (a fall in demand from the rest of the world). The governments of both countries care about their external balance b , which is given by:

$$b = \rho (g^* - g) - u \text{ and } b^* = \rho (g - g^*) - u$$

where $\rho > 0$.

Each country chooses fiscal policy in order to minimize a loss function: $L = \omega Y^2 + b^2$ and symmetrically $L^* = \omega^* Y^{*2} + b^{*2}$ where $\omega, \omega^* > 0$ represent the home and foreign weights of income relative to the external balance in the loss function.

1. Show that the optimum policy for the home country when determining policy independently is

$$g = \frac{(\rho^2 - \omega\phi\psi)g^* + (\omega\phi - \rho)u}{\omega\phi^2 + \rho^2}$$

2. Show that the uncoordinated (Nash) equilibrium is:

$$g = g^* = \frac{\omega\phi - \rho}{\omega\phi(\phi + \psi)} u$$

$$y = y^* = -\frac{\rho u}{\omega\phi}$$

$$b = -u$$

$$L = \left(1 + \frac{\rho^2}{\omega\phi^2}\right) u^2$$

3. Show that the loss incurred when $g = g^* = 0$ is $L = L^* = (1 + \omega) u^2$. When is inaction preferable to action?
4. Governments now cooperate and set fiscal policy as if they were a single country. What is the outcome? Does it improve in comparison to the uncoordinated equilibrium?